

Extended axion electrodynamics: Anomalous dynamo-optical response induced by gravitational pp-waves

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We extend the Einstein-Maxwell-axion theory including into the Lagrangian cross-terms of the dynamo-optical type, which are quadratic in the Maxwell tensor, linear in the covariant derivative of the macroscopic velocity four-vector, and linear in the pseudoscalar (axion) field or its gradient four-vector. We classify the new terms with respect to irreducible elements of the covariant derivative of the macroscopic velocity four-vector of the electromagnetically active medium: the expansion scalar, acceleration four-vector, shear and vorticity tensors. Master equations of the extended axion electrodynamics are used for the description of the response of an axionically active electrodynamic system, induced by a pp-wave gravitational background. We show that this response has a critical character, i.e., the electric and magnetic fields, dynamo-optically coupled to the axions, grow anomalously under the influence of the external pp-wave gravitational field.

I. INTRODUCTION

Axion electrodynamics as an extension of the Faraday - Maxwell electromagnetic theory is based on the prediction of axions, massive pseudo-Goldstone bosons [1–3]. The first discussion concerning the pseudoscalar-photon interaction appeared in [4]; however, the interest to axion electrodynamics has grown significantly later, after publication of the paper [5]. Important aspects of the axion theory and of its astrophysical and cosmological applications can be found, e.g., in reviews and book [6–10]. Results of experimental investigations of the axion-photon interactions are published, e.g., in [11–16]).

The axion electrodynamics can be indicated as the standard one, when the Lagrangian contains only one cross-term $\frac{1}{4}\phi F_{mn}^* F^{mn}$, in which the product of the pseudoscalar (axion) field ϕ and of dual Maxwell (pseudo)tensor behaves as a true tensor. In that theory axion-induced phenomena can be visualized, when the axion field ϕ has non-vanishing gradient four-vector $\nabla_k \phi \neq 0$; respectively, the invariant $I = g^{ik} \nabla_i \phi \nabla_k \phi$ can be positive, negative or equal to zero. The last case relates to models with pp-wave symmetry, for which the axion field depends on the retarded time only. Here we focus just on this model and consider the pp-wave gravitational background as a scene for evolution of the electromagnetic field in a non-uniformly moving axionically active medium.

In the papers [17–23] we considered extensions of the axion electrodynamics, keeping in mind that even if the axion field is initially constant, external gravitational and electromagnetic fields are able to activate frozen axion-photon couplings in course of evolution of the corresponding physical system. In particular, we considered a non-minimal axion-photon coupling activated by gravitational waves [17]; non-stationary optical activity and

gradient-type models of the axion-photon coupling in a cosmological context [18, 19]; fingerprints of dark matter axions in the terrestrial electric and magnetic field variations [20]; electromagnetic waves in an axion-active plasma [21, 22]; axionically induced anomalous behavior of the electromagnetic response to the gravitational wave action [23].

In this work we consider the so-called *dynamo-optical extension* of the axion electrodynamics. This term was introduced in [24] to describe the influence of a non-uniform (irregular) motion of a physical system on its electromagnetic response. The generalization of dynamo-optical type of the Einstein-Maxwell theory was carried out in [25]. In this paper we add a new element into the theory, namely, the pseudoscalar (axion) field, thus providing the dynamo-optical extension of the axion electrodynamics, which is a distinctive part of the Einstein-Maxwell-axion theory.

II. THE MODEL

A. Action functional and standard definitions

Master equations of the extended axion electrodynamics form the sub-set of the total system of equations of the Einstein - Maxwell - axion model (see, e.g., [17, 23]); they can be obtained by variation of the action functional

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{2\kappa} + L_{(m)} + L_{(EM)} + L_{(A)} \right] \quad (1)$$

with respect to the electromagnetic potential four-vector A_i , and dimensionless pseudoscalar (axion) field ϕ , respectively. Equations of the gravity field, as a result of variation of this action functional with respect to space-time metric g_{ik} , are presented in [22] and we do not consider them in this paper. As usual, g is the determinant of the metric; R is the Ricci scalar; $\kappa = \frac{8\pi G}{c^4}$ is the Einstein constant. The Lagrangian of the axion field is of

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the form

$$L_{(A)} = \frac{1}{2} \Psi_0^2 \left[m_{(A)}^2 \phi^2 + V(\phi^2) - g^{mn} \nabla_m \phi \nabla_n \phi \right], \quad (2)$$

where ∇_m is a covariant derivative; the parameter $\frac{1}{\Psi_0}$ is a coupling constant of the axion-photon interaction; the term $m_{(A)} = \frac{c}{\hbar} m_{(\text{axion})}$ is a re-scaled axion mass $m_{(\text{axion})}$; \hbar is the Planck constant. The total Lagrangian of the electromagnetic field in the axionically active medium, $L_{(\text{EM})}$, is considered to be quadratic with respect to the Maxwell tensor $F_{ik} \equiv \nabla_i A_k - \nabla_k A_i$. This Lagrangian includes the dual tensor $F^{*mn} = \frac{1}{2} \epsilon^{mnpq} F_{pq}$, where, as usual, $\epsilon^{mnpq} \equiv \frac{1}{\sqrt{-g}} E^{mnpq}$ is the Levi-Civita tensor, E^{mnpq} is the skew-symmetric Levi-Civita symbol with $E^{0123}=1$. The dual Maxwell tensor satisfies the condition $\nabla_k F^{*ik}=0$, which is treated as a sub-set of electrodynamic equations.

Keeping in mind the necessity of a phenomenological decomposition of the extended Lagrangian $L_{(\text{EM})}$ into irreducible parts, we use the following standard representation of the tensor F^{ik} :

$$F^{ik} = E^i U^k - E^k U^i - \epsilon^{ikmn} B_m U_n. \quad (3)$$

Here U^i is the macroscopic velocity four-vector; we use the Landau-Lifshitz definition of U^i considering it as a time-like eigen-vector of the stress-energy tensor of the matter (see, e.g., [23] for details). The electric field four-vector, $E^i \equiv F^{ik} U_k$, and the magnetic induction four-vector, $B_i \equiv F_{ik}^* U^k$, are, clearly, orthogonal to the velocity four-vector U^i .

In order to classify irreducible terms describing interactions of the dynamo-optical type, we use in the decomposition of the Lagrangian $L_{(\text{EM})}$ the following standard representation of the covariant derivative of the velocity four-vector:

$$\nabla_i U_k = U_i D U_k + \sigma_{ik} + \omega_{ik} + \frac{1}{3} \Delta_{ik} \Theta. \quad (4)$$

Here $D U_k \equiv U^i \nabla_i U_k$ is the medium acceleration four-vector; $\Theta \equiv \nabla_m U^m$ is the expansion scalar; the traceless symmetric shear tensor σ_{ik} , the skew-symmetric vorticity tensor ω_{ik} , and the projector Δ_{ik} are given by the formulas

$$\sigma_{ik} \equiv \Delta_{(i}^p \Delta_{k)}^q \nabla_p U_q - \frac{1}{3} \Delta_{ik} \Theta,$$

$$\omega_{ik} \equiv \Delta_{[i}^p \Delta_{k]}^q \nabla_p U_q, \quad \Delta_{ik} = g_{ik} - U_i U_k. \quad (5)$$

The symbols (ik) and $[ik]$ define the operations of symmetrization and skew-symmetrization, respectively. Also, we use two auxiliary tensors: the angular velocity (pseudo) four-vector ω_i , and the skew-symmetric tensor Ω_{pq} given by

$$\omega_i \equiv -\omega_{ik}^* U^k, \quad \Omega_{pq} \equiv U_{[p} D U_{q]}. \quad (6)$$

We consider the quantities $\phi, \nabla_i \phi, E^i, B^k, U^i, D U_i, \Theta, \sigma_{ik}, \omega_{ik}$ to be basic elements of the decomposition of the electromagnetic field Lagrangian.

B. Decomposition of the term $L_{(\text{EM})}$

We divide the electromagnetic field Lagrangian $L_{(\text{EM})}$ into seven parts

$$L_{(\text{EM})} = L_0 + L_1 + L_2 + L_3 + L_4 + L_5 + L_6. \quad (7)$$

For our purposes it is convenient to write the term L_0 in the following form

$$L_0 = \frac{1}{2\mu} (n^2 E^i E_i - B^i B_i) + \phi B^i E_i, \quad (8)$$

using the (E, B) -representation. Here $n^2 \equiv \epsilon\mu$, i.e., n is the refraction index of the medium with dielectric and magnetic permittivities ϵ and μ , respectively. When $\epsilon=1, \mu=1$, the term (8) is of the form $\frac{1}{4} (F^{mn} F_{mn} + \phi F^{mn} F_{mn}^*)$, thus recovering the Lagrangian of the standard axion electrodynamics.

The term L_1 does not contain the axion field ϕ , however, in contrast to (8), it is linear in the covariant derivative of the velocity four-vector and contains six irreducible scalar terms equipped by six phenomenological constants $\lambda_{11}, \dots, \lambda_{16}$ [25]:

$$\begin{aligned} L_1 = & \frac{1}{4} \Theta (\lambda_{11} E_k E^k + \lambda_{12} B_k B^k) + \\ & + \frac{1}{4} \sigma^{km} (\lambda_{13} E_k E_m + \lambda_{14} B_k B_m) + \\ & + \frac{1}{4} \epsilon^{ikmn} E_i B_k (\lambda_{15} \Omega_{mn} + \lambda_{16} \omega_{mn}). \end{aligned} \quad (9)$$

Terms linear in the pseudoscalar field ϕ and in the convective derivative $D\phi$ contain pseudovector B^i providing L_2 and L_3 to be pure scalars:

$$L_2 = \frac{\phi}{4} E_m B_n (\lambda_{21} \Theta g^{mn} + \lambda_{22} \sigma^{mn} + \lambda_{23} \omega^{mn}), \quad (10)$$

$$L_3 = \frac{D\phi}{4} E_m B_n (\lambda_{31} \Theta g^{mn} + \lambda_{32} \sigma^{mn} + \lambda_{33} \omega^{mn}). \quad (11)$$

All terms linear in the spatial gradient of the pseudoscalar field, $\Delta_k^i \nabla_k \phi$, can be reduced to the set of three scalars L_4, L_5, L_6 . First, we construct scalars linear in the vorticity tensor; it is convenient to use in this case the pseudovector ω_i guaranteeing the product $\omega_m \nabla_n \phi$ to be a pure tensor:

$$\begin{aligned} L_4 = & \frac{1}{4} \omega_{(m} \nabla_{n)} \phi [\Delta^{mn} (\lambda_{41} E_k E^k + \lambda_{42} B_k B^k) + \\ & + (\lambda_{43} E^m E^n + \lambda_{44} B^m B^n)] . \end{aligned} \quad (12)$$

Second, we list the terms linear in the acceleration four-vector $D U_k$ (terms with ϕ can not appear):

$$L_5 = \frac{1}{4} \nabla_n \phi D U_m (\lambda_{51} \Delta^{mn} E^k B_k +$$

$$+\lambda_{52}E^n B^m + \lambda_{53}E^m B^n) . \quad (13)$$

Third, there are two terms linear in the shear tensor

$$L_6 = \frac{1}{4} \eta^{nmp} \sigma_{mk} \nabla_n \phi (\lambda_{61} E^k E_p + \lambda_{62} B^k B_p) . \quad (14)$$

Our statement is that the decomposition presented above is irreducible, and the number of independent coupling constants $\lambda_{11}, \dots, \lambda_{62}$, appeared in front of listed terms, is twenty one. Keeping in mind symmetry motives, further we reduce the number of these parameters using the following relationships:

$$\begin{aligned} \lambda_{13} &= -\lambda_{12}, \quad \lambda_{13} = \lambda_{14}, \quad \lambda_{21} = \frac{1}{3} \lambda_{22}, \quad \lambda_{31} = \frac{1}{3} \lambda_{32}, \\ \lambda_{42} &= -\lambda_{41}, \quad \lambda_{44} = \lambda_{33}, \quad \lambda_{62} = \lambda_{61}. \end{aligned} \quad (15)$$

Also, we put $\lambda_{16}=0$, since the corresponding term does not enter the electrodynamic equations due to their specific structure.

C. Master equations

1. A pp-wave gravitational background

We consider test electromagnetic and pseudoscalar fields in the pp-wave gravitational background with the line element

$$ds^2 = 2dudv - L^2 [e^{2\beta}(dx^2)^2 + e^{-2\beta}(dx^3)^2], \quad (16)$$

where $u = \frac{ct-x^1}{\sqrt{2}}$ is the retarded time and $v = \frac{ct+x^1}{\sqrt{2}}$ is the advanced time. For such model the background factor $L(u)$ satisfies the requirements

$$L'' + (\beta')^2 L = 0, \quad L(0)=1, \quad L'(0)=0, \quad (17)$$

where the prime denotes the derivative with respect to the retarded time u , and $\beta(u)$ is arbitrary function of the retarded time with initial value $\beta(0)=0$. We consider the medium to be at rest, and the velocity four-vector to be of the form $U_u=U_v=\frac{1}{\sqrt{2}}$, $U_2=U_3=0$. For this case $DU_i=0$, $\omega_{ik}=0$, $\Theta=\frac{\sqrt{2}L'}{L}$,

$$\sigma_i^k = \frac{\Theta}{2} \left(\frac{1}{3} \Delta_i^k - \delta_i^1 \delta_1^k \right) + \frac{\beta'}{\sqrt{2}} (\delta_i^2 \delta_2^k - \delta_i^3 \delta_3^k). \quad (18)$$

We assume that the electromagnetic and axion fields inherit the pp-wave symmetry of the gravitational background, thus, the unknown functions depend on u only: $E^i(u)$, $B^k(u)$, $\phi(u)$ (see [23]).

2. Electrodynamic equations

Electrodynamic equations have the standard form

$$\nabla_k H^{ik} = 0, \quad \nabla_k F^{*ik} = 0, \quad (19)$$

where the excitation tensor can be now written as

$$H^{ik} = U^n \left[\delta_{mn}^{ik} \frac{\partial L_{(EM)}}{\partial E_m} + \epsilon_{mn}^{ik} \frac{\partial L_{(EM)}}{\partial B_m} \right], \quad (20)$$

using the explicit (E, B) - representation of the scalar $L_{(EM)}$ (7)-(14). This procedure is routine, and we omit details of the H^{ik} decomposition. When unknown functions depend on retarded time only, integration of (19) gives six integrals

$$L^2 H^{iu}(u) = H^{iu}(0), \quad L^2 F^{*iu}(u) = F^{*iu}(0), \quad (21)$$

($i=v, x^2, x^3$). Three of them do not contain $\phi(u)$:

$$B_v(u) = \frac{B_v(0)}{L^2(u)}, \quad B_2(u) = e^{2\beta} [B_2(0) + E_3(u) - E_3(0)],$$

$$B_3 = e^{-2\beta} [B_3(0) + E_2(0) - E_2(u)]. \quad (22)$$

The longitudinal integral (for $i=v$) yields

$$E_v(u) = \frac{\varepsilon E_v(0) - B_v(0)[\phi(u) - \phi(0)]}{L^2 [\varepsilon + \frac{1}{6} \Theta(u)(3\lambda_{11} - \lambda_{13})]}. \quad (23)$$

Two remaining integrals contain both $\phi(u)$ and $\phi'(u)$; using (22) they can be written in the form:

$$2\mathcal{A}(u)e^{-2\beta}E_2(u) + 3\beta'\mathcal{B}(u)E_3(u) = \mathcal{J}_2(u),$$

$$3\beta'\mathcal{B}(u)E_2(u) + 2\mathcal{A}(u)e^{2\beta}E_3(u) = \mathcal{J}_3(u), \quad (24)$$

where the auxiliary functions are defined as follows:

$$\mathcal{A}(u) = \lambda_{13}\Theta + 6 \left(\varepsilon - \frac{1}{\mu} \right),$$

$$\mathcal{B}(u) = \lambda_{32}\phi' + \sqrt{2}\lambda_{22}\phi. \quad (25)$$

The source-like term $\mathcal{J}_2(u)$ in (24) is of the form

$$\begin{aligned} \mathcal{J}_2(u) &= \frac{3}{2\sqrt{2}} [E_3(0) - B_2(0)] \{ 6\lambda_{22}\beta'\phi + \Theta\mathcal{B} + \\ &+ \sqrt{2}\beta'\phi'(\lambda_{32} - 2\lambda_{61}) + 48\sqrt{2}[\phi - \phi(0)] \} + \\ &+ [B_3(0) + E_2(0)] \left\{ e^{-2\beta} \left[\Theta(\lambda_{13} - 6\lambda_{11}) - 3\sqrt{2}\lambda_{13}\beta' \right] \right. \\ &\left. + \frac{12}{\mu} (1 - e^{-2\beta}) \right\} + 12E_2(0) \left(\varepsilon - \frac{1}{\mu} \right). \end{aligned} \quad (26)$$

The term \mathcal{J}_3 can be obtained from (26) by replacements $\beta \rightarrow -\beta$, $E_2(0) \rightarrow E_3(0)$, $B_2(0) \rightarrow -B_3(0)$.

3. Equation of the axion field evolution

Variation of the action functional (1) with respect to the axion field ϕ gives the equation

$$\left[\square + m_{(A)}^2 + V'(\phi^2) \right] \phi = -\frac{1}{\Psi_0^2} (B_k E^k - \mathcal{J}). \quad (27)$$

When $\mathcal{J}=0$, we deal with standard equation for the pseudoscalar field; using explicit representation of L_2, \dots, L_6 (see (10)-(14)) we can write the dynamo-optical source \mathcal{J} in the following compact form:

$$\mathcal{J} = 2 \left\{ - \left(\frac{L_2}{\phi} \right) + (\Theta + D) \left(\frac{L_3}{D\phi} \right) + \nabla_n \left[\frac{\partial}{\partial(\nabla_n \phi)} (L_4 + L_5 + L_6) \right] \right\}. \quad (28)$$

Clearly, this term is quadratic in the components of the electromagnetic field, E^i and B^k ; it is up to second order with respect to irreducible elements of the decomposition of the covariant derivative of the velocity four vector, Θ , DU_k , σ_{mn} , ω_{mn} , and contains second covariant derivatives $\nabla_s \nabla_m U_n$.

4. Initial state

When the gravitational wave is absent ($u < 0$), the pseudoscalar, electric and magnetic fields are considered to be constant ($\phi(0)$, $E_i(0)$, $B_i(0)$, respectively), and the system as a whole to be at rest. Clearly, at $u=0$ the equations (22) - (24) with (25) and (26) convert into identities for arbitrary $\phi(0)$, $E_i(0)$, $B_i(0)$. The initial value $\phi(0)$ can be found from the reduced equation (27) with $\mathcal{J}=0$:

$$\left[m_{(A)}^2 + V'(\phi^2(0)) \right] \phi(0) = -\frac{1}{\Psi_0^2} B_k(0) E^k(0), \quad (29)$$

the values $E_i(0)$, $B_i(0)$ being arbitrary constants.

III. ANOMALOUS BEHAVIOR OF THE ELECTROMAGNETIC RESPONSE

The presented above set of master equations of the axion electrodynamics in the pp-wave gravitational background (see (22)-(28)) is a coupled system of nonlinear equations; search for exact solution of this system is a sophisticated but very interesting problem. In this sense the exact solutions discussed in [23] relate to the case of absence of the dynamo-optic phenomena, nevertheless, that results let us expect the formulated dynamo-optical problem to be solved in the nearest future with some special conditions for the coupling constants. Here we restrict our-selves by analysis of a truncated model, which, nevertheless, displays the main new feature: the anomalous character of the axion-photon coupling in the medium moving non-uniformly.

A. Longitudinal electromagnetic fields

We use the term *longitudinal*, when the initial electric and/or magnetic fields are directed along the axis of the gravitational wave propagation. In this case transversal components can not be produced, thus, we deal with magnetic and electric fields given by (22), (23), and with the axion field of the form:

$$\phi(u) = \frac{\phi(0) [\varepsilon m_{(A)}^2 \Psi_0^2 + 2B_v^2(0)]}{L^4 m_{(A)}^2 \Psi_0^2 [\varepsilon + \frac{1}{6} \Theta (3\lambda_{11} - \lambda_{13})] + 2B_v^2(0)},$$

$$\phi(0) = \frac{2E_v(0)B_v(0)}{m_{(A)}^2 \Psi_0^2}, \quad (V(\phi^2) \equiv 0). \quad (30)$$

The denominator of $E_v(u)$ in (23) can tend to zero value at $u \rightarrow u^*$, when $\Theta(u^*) = 6\varepsilon(\lambda_{13} - 3\lambda_{11})^{-1}$, thus providing an anomalous growth of the longitudinal electric response. The axion field at this moment, $\phi(u^*)$, remains finite, when $B_v(0) \neq 0$.

B. Transversal electromagnetic fields

When $E_v(0)=B_v(0)=0$, but $E_2(0), \dots, B_3(0) \neq 0$, we deal with three-dimensional nonlinear system of coupled evolutionary equations. Let us illustrate the appearance of the response anomaly for the case of relic dark matter axion domination (see, e.g., [19, 20]). This term means that the density of axions, produced by electromagnetic field, is much smaller than the density of relic cosmological axions. In such case we consider the function $\phi(u)$ to be fixed, and face with linear algebraic system (24). The corresponding Cramer's determinant $\Delta(u)$

$$\Delta(u) = 4\mathcal{A}^2(u) - 9\beta'^2 \mathcal{B}^2(u)$$

takes zero value at the moment u_* , for which $\mathcal{A}(u_*) = \pm \frac{3}{2} \beta' \mathcal{B}(u_*)$, or in more details

$$\Theta(u_*) = \frac{6}{\lambda_{13}} \left[\left(\frac{1}{\mu} - \varepsilon \right) \pm \frac{\beta'}{4} \left(\lambda_{32} \phi' + \sqrt{2} \lambda_{22} \phi \right) \right]_{|u_*}.$$

This anomaly, appeared in the response of transversal electric and magnetic fields, is mixed and can be indicated as axionic-dynamo-optical, in contrast to pure dynamo-optical longitudinal anomaly in (23).

IV. CONCLUSIONS

1. Dynamo - optical extension of the Einstein - Maxwell - axion theory is shown to have twenty one coupling constants as a maximum; the number of essential parameters can be reduced to thirteen by physical assumptions.
2. Longitudinal electric and magnetic fields in an axionically active medium, evolving in the pp-wave gravitational background, can possess dynamo - optical

anomaly, which describes an amplification of the electromagnetic response, when $3\lambda_{11} - \lambda_{13} \neq 0$.

3. Transversal electric and magnetic fields can display an anomaly of a new type, which differs from the dynamo-optical one by the dependence on the axion field.

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